

Correlated stability conjecture revisited

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Abstract

Correlated stability conjecture (CSC) proposed by Gubser and Mitra [1,2] linked the thermodynamic and classical (in)stabilities of black branes. The classical instabilities, whenever occurring, were conjectured to arise as Gregory-Laflamme (GL) instabilities of translationally invariant horizons. In [3] it was shown that the thermodynamic instabilities, specifically the negative specific heat, indeed result in the instabilities in the hydrodynamic spectrum of holographically dual plasma excitations. A counter-example of CSC was presented in the context of black branes with scalar hair undergoing a second-order phase transition [4]. In this paper we discuss a related counter-example of CSC conjecture, where a thermodynamically stable translationally invariant horizon has a genuine tachyonic instability. We study the spectrum of quasinormal excitations of a black brane undergoing a continuous phase transition, and explicitly identify the instability. We compute the critical exponents of the critical momenta and the frequency of the unstable fluctuations and identify the dynamical critical exponent of the model.

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Contents

1	Introduction	2
2	Thermodynamics and hydrodynamics of exotic black branes	5
3	GL tachyons of exotic black branes	8

1 Introduction

Black holes with translationally invariant horizons (also referred to as black branes) are ubiquitous in string theory [5]. They are particularly important in the context of holographic gauge theory/string theory correspondence [6], where they describe the thermal equilibrium states of the dual gauge theory plasma. In case of $d = p + 1$ space-time dimensional non-conformal gauge theory plasma, the dual black branes of the $(d + 1)$ -dimensional supergravity are rather complicated, and have multiple scalar hair, see for example [7, 8]. Being dual to a finite temperature gauge theory plasma, these black branes have nonvanishing Hawking temperature and necessarily are non-supersymmetric. Thus, their classical stability is not assured. The linear stability analysis of black branes is complicated by mixing the scalar hair and the metric fluctuations. A much simpler exercise is to compute the thermodynamic potentials of the black branes, *i.e.*, the free energy density, the energy density, etc., and analyze a thermodynamic stability of the resulting potentials. It was a very welcome conjecture by Gubser and Mitra [1, 2] — *Correlated Stability Conjecture* — that identified the thermodynamic stabilities or instabilities of translationally invariant horizons with the classical stabilities or instabilities correspondingly. Moreover, the classical instabilities, whenever occurring, were conjectured to appear as Gregory-Laflamme instabilities [9, 10]. Recall that GL instabilities of the translationally invariant horizons are linearized fluctuations $\delta\Phi \propto e^{-i\omega t + i\vec{k}\cdot\vec{x}}$ with a dispersion relation $(\mathfrak{w}, \mathfrak{q})$ such that

$$\left. \text{Im}(\mathfrak{w}) \right|_{\mathfrak{q} < \mathfrak{q}_c} > 0, \quad (1.1)$$

where

$$\mathfrak{w} \equiv \frac{\omega}{2\pi T}, \quad \mathfrak{q} \equiv \frac{|\vec{k}|}{2\pi T}, \quad (1.2)$$

and T is the temperature. Note that we always assume that $\text{Im}(\mathfrak{q}) = 0$, *i.e.*, fluctuation amplitudes do not grow exponentially along the translational directions of a horizon.

A standard claim in classical thermodynamics¹ is that a system is thermodynamically stable if the Hessian $\mathbb{H}_{s,Q_A}^{\mathcal{E}}$ of the energy density $\mathcal{E} = \mathcal{E}(s, Q_A)$ with respect to the entropy density s and charges $Q_A \equiv \{Q_1, \dots, Q_n\}$, *i.e.*,

$$\mathbb{H}_{s,Q_A}^{\mathcal{E}} \equiv \begin{pmatrix} \frac{\partial^2 \mathcal{E}}{\partial s^2} & \frac{\partial^2 \mathcal{E}}{\partial s \partial Q_B} \\ \frac{\partial^2 \mathcal{E}}{\partial Q_A \partial s} & \frac{\partial^2 \mathcal{E}}{\partial Q_A \partial Q_B} \end{pmatrix}, \quad (1.3)$$

does not have negative eigenvalues. In the simplest case $n = 0$, *i.e.*, no conserved charges, the thermodynamic stability implies that

$$0 < \frac{\partial^2 \mathcal{E}}{\partial s^2} = \frac{T}{c_v}, \quad (1.4)$$

that is the specific heat c_v is positive. In the context of gauge theory/string theory correspondence black holes with translationary invariant horizons in asymptotically anti-de-Sitter space-time are dual (equivalent) to equilibrium thermal states of certain strongly coupled systems. Thus, the above thermodynamic stability criteria should be directly applicable to black branes as well. CSC asserts that it is only when the Hessian (1.3) for a given black brane geometry is positive, the spectrum of on-shell excitations in this background geometry is free from tachyons.

CSC is trivially true in a holographic context, whenever the specific heat of the corresponding black brane is negative [3]. Indeed, in the absence of conserved charges, the speed of hydrodynamic (sound channel) modes in the dual plasma is

$$c_s^2 = \frac{s}{c_v}, \quad (1.5)$$

where s, \mathcal{E} are the entropy and the energy densities, and c_v

$$c_v = \left(\frac{\partial \mathcal{E}}{\partial T} \right)_v, \quad (1.6)$$

is the specific heat. Thus whenever $c_v < 0$, the sound modes in plasma, and correspondingly the dual quasinormal modes in the gravitational background, are unstable².

The situation is more complicated in holographic examples in the presence of conserved charges. Consider a strongly coupled $\mathcal{N} = 4$ $SU(N)$ supersymmetric Yang-Mills theory in the planar limit at finite temperature T and a chemical potential μ for a single global $U(1) \subset SO(6)$ R-symmetry. The dual gravitational geometry is that of the

¹Assuming that the temperature is positive.

²Such instability occurs in $\mathcal{N} = 2^*$ holographic plasma [7, 11].

Reissner-Nordström asymptotic AdS_5 black brane [12–14]. For any temperature larger than the critical one, $T_c = \mu\sqrt{2}/\pi$, there are two phases of the plasma. Although one of the phases has a negative specific heat, the sound modes in plasma are always stable [15]:

$$c_v = 4\pi^2 N^2 T^3 \frac{(1+\kappa)(3-\kappa)}{(2+\kappa)^2(2-\kappa)}, \quad \mathfrak{w} = \pm \frac{1}{\sqrt{3}} \mathfrak{q} - \frac{i}{3} \frac{\kappa+2}{2\kappa+2} \mathfrak{q}^2 + \mathcal{O}(\mathfrak{q}^3), \quad (1.7)$$

where

$$\frac{2\pi T}{\mu} = \sqrt{\kappa} + \frac{2}{\sqrt{\kappa}}, \quad (1.8)$$

so that given $T > T_c$ there are two possible values of κ distinguishing two different phases: one with $\kappa < 2$ (a thermodynamically stable phase) and one with $\kappa > 2$ (a thermodynamically unstable phase). There is no contradiction here with the argument presented in [3] — the speed of sound in the presence of conserved charges takes a more complicated form than in (1.5). In case of a charged plasma we have

$$c_s^2 = \left((\mathcal{E} + P) \frac{\partial(P, \rho)}{\partial(T, \mu)} + \rho \frac{\partial(\mathcal{E}, P)}{\partial(T, \mu)} \right) \left((\mathcal{E} + P) \frac{\partial(\mathcal{E}, \rho)}{\partial(T, \mu)} \right)^{-1}, \quad (1.9)$$

where P is the pressure. Remarkably, there is no contradiction with CSC conjecture as well! Indeed, it was shown in [15] that a two-point retarded correlation function of the charge density fluctuations has a pole (and thus the SYM plasma has a physical excitation with this dispersion relation) at

$$\mathfrak{w} = \mathcal{A} i(\kappa - 2) \mathfrak{q}^2 + \mathcal{O}(\mathfrak{q}^4), \quad \mathcal{A} = 0.33339(2), \quad (1.10)$$

so that a phase with $\kappa > 2$ is classically unstable, at least with respect to fluctuations with sufficiently small \mathfrak{q} . We emphasize that even though the gravitational instability is in the quasinormal sound channel mode, it is not a hydrodynamic sound, which is characterized by the dispersion relation $\mathfrak{w} \propto \mathfrak{q}$, see (1.7).

So far, the examples we discussed provide support for CSC conjecture. It is believed however, that CSC conjecture is false [4]³. The idea advocated in [4] is to consider gravitational backgrounds with translationally invariant horizon that are dual to gauge theory plasmas undergoing a continuous phase transition. In the vicinity of the phase transition the condensate does not noticeably modify the thermodynamics, and thus should not affect the thermodynamic stability of the system. On the other hand, the

³See also [16].

phase of the system with the higher free energy is expected to be classically unstable. The condensation of the tachyon should bring the system to the equilibrium phase with the lowest free energy. The tachyon of the unstable plasma phase should appear in a dual gravitational description as a tachyonic quasinormal mode, *i.e.*, the GL instability (1.1).

In this paper we discuss an explicit example of the physical scenario suggested in [4] from the perspective of the quasinormal spectrum. We identify the GL tachyon and compute the critical exponents for the critical momentum and the frequency of the corresponding fluctuations. Our model is the 'exotic hairy black hole' introduced in [17]. The dispersion relation for the sound waves in this model was studied in [18]. Much like in case of the R-charged $\mathcal{N} = 4$ SYM plasma, there are no instabilities in hydrodynamic modes. We review the thermodynamics and the hydrodynamics of the model in section 2. In section 3 we identify non-hydrodynamic quasinormal excitations in the sound channel of the gravitational background with the dispersion features identical to those of the GL instabilities (1.1). The GL tachyons exist in the symmetry broken phase only, and disappear from the spectrum at the second-order phase transition. We compute critical exponents associated with vanishing of \mathbf{q}_c and $\mathbf{w}_c \equiv \mathbf{w}(\mathbf{q} = 0)$ at the transition. Finally, we comment on the dynamical universality class of the model.

2 Thermodynamics and hydrodynamics of exotic black branes

The model considered here is not a string theory derived example of gauge/gravity correspondence — rather, it should be viewed as a phenomenological model of holography. Of course, whether or not the model can be embedded into the full string theory is irrelevant in so far as one is interested in CSC. The holography here is simply a useful tool to think physically about mathematical problem of (in)stabilities of translationally invariant horizons.

Following [19], consider a relativistic conformal field theory in 2+1 dimensions, deformed by a relevant operator \mathcal{O}_r :

$$H_{CFT} \rightarrow \tilde{H} = H_{CFT} + \lambda_r \mathcal{O}_r. \quad (2.1)$$

Such a deformation softly breaks the scale invariance and induces the renormalization group flow. We further assume that the deformed theory \tilde{H} has an irrelevant operator \mathcal{O}_i that mixes along the RG flow with \mathcal{O}_r . The explicit holographic model realizing

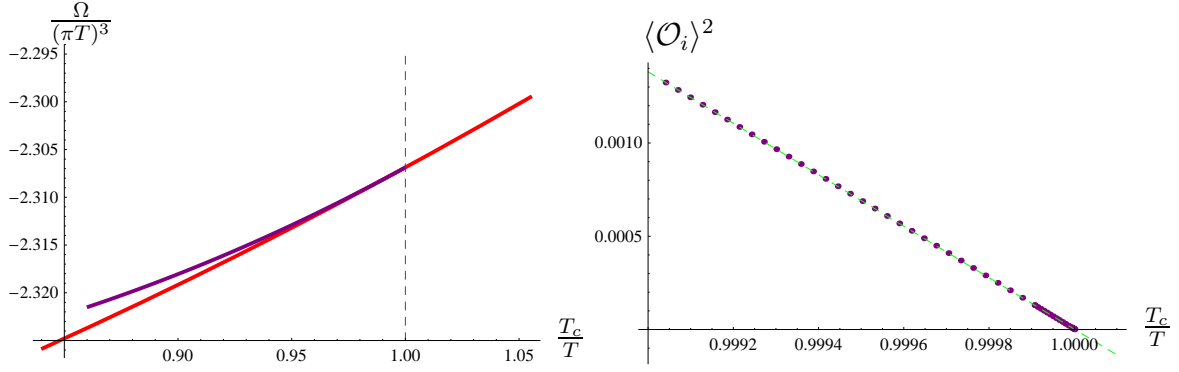


Figure 1: (Colour online) The free energy densities Ω_0 of the symmetric phase (red curve, left plot) and Ω_d of the symmetry broken phase (purple curve, left plot) as a function of the reduced temperature $\frac{T}{T_c}$ in the gauge theory plasma dual to the holographic RG flow in [17]. The right plot represents the square of $\langle \mathcal{O}_i \rangle$ (which we use as an order parameter for the transition) as a function of the reduced temperature. The dashed green line is a linear fit to $\langle \mathcal{O}_i \rangle^2$.

this scenario was discussed in [17]:

$$S_4 = S_{CFT} + S_r + S_i = \frac{1}{2\kappa^2} \int dx^4 \sqrt{-\gamma} [\mathcal{L}_{CFT} + \mathcal{L}_r + \mathcal{L}_i], \quad (2.2)$$

$$\mathcal{L}_{CFT} = R + 6, \quad \mathcal{L}_r = -\frac{1}{2} (\nabla \phi)^2 + \phi^2, \quad \mathcal{L}_i = -\frac{1}{2} (\nabla \chi)^2 - 2\chi^2 - g\phi^2\chi^2, \quad (2.3)$$

where we split the action into (a holographic dual to) a CFT part S_{CFT} ; its deformation by a relevant operator \mathcal{O}_r ; and a sector S_i involving an irrelevant operator \mathcal{O}_i along with its mixing with \mathcal{O}_r under the RG dynamics. The four dimensional gravitational constant κ is related to the central charge c of the UV fixed point as

$$c = \frac{192}{\kappa^2}. \quad (2.4)$$

In our case the scaling dimension of \mathcal{O}_r is 2 and the scaling dimension of \mathcal{O}_i is 4. In order to have asymptotically AdS_4 solutions, we assume that only the normalizable mode of \mathcal{O}_i is nonzero near the boundary. Finally, we assume that $g < 0$ in order to holographically induce the critical behavior [19].

Effective action (2.2) has a $\mathbb{Z}_2 \times \mathbb{Z}_2$ discrete symmetry that acts as a parity transformation on the scalar fields ϕ and χ . The discrete symmetry $\phi \rightarrow -\phi$ is softly broken by a relevant deformation of the AdS_4 CFT; while the $\chi \rightarrow -\chi$ symmetry is broken

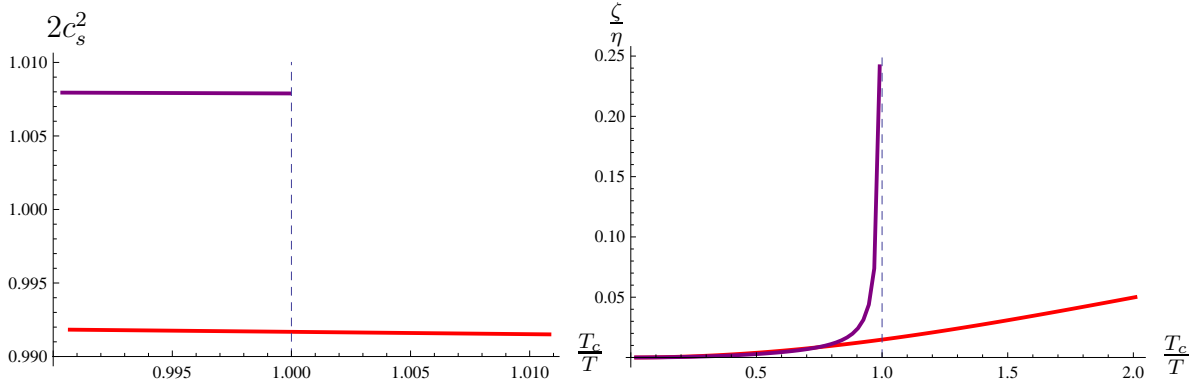


Figure 2: (Colour online) The speed of sound c_s and the ratio of bulk-to-shear viscosities in the gauge theory plasma dual to the holographic RG flow in [17] as a function of the reduced temperature $\frac{T_c}{T}$.

spontaneously. Spontaneous breaking of the latter symmetry is caused by the development of the condensate for the irrelevant operator \mathcal{O}_i . The unusual part of this phase transition is that the symmetry broken phase occurs at high temperatures (rather than at low temperatures) and that the broken phase has a higher free energy density than the unbroken phase with $\langle \mathcal{O}_i \rangle = 0$. The free energy densities of the \mathbb{Z}_2 -symmetric phase ($\langle \mathcal{O}_i \rangle = 0$) and the symmetry broken phase ($\langle \mathcal{O}_i \rangle \neq 0$) are presented in Figure 1.

The hydrodynamic sound channel quasinormal modes of both phases were studied in details in [18]. There is a jump discontinuity at the transition in the speed of sound; with the speed of sound in the symmetry broken phase being $\sim 1\%$ higher than in the symmetric phase. Both the shear and the bulk viscosities in the two phases are positive. Figure 2 collects the results for the speed of sound and the ratio of bulk-to-shear viscosities in the exotic plasma. In the symmetry broken phase the bulk viscosity diverges in the vicinity of the phase transition [18]⁴

$$\left. \frac{\zeta}{\eta} \right|_{broken} \propto |\langle \mathcal{O}_i \rangle|^{-2} \propto (T - T_c)^{-1}. \quad (2.5)$$

Note that since $c_s^2 > 0$, the specific heat (see (1.5)) of both the symmetric and the symmetry broken phases is positive — the two phases are thermodynamically stable⁵.

⁴The shear viscosity is continuous and finite at the transition: $\eta/s = 1/4\pi$ [20–22].

⁵We also verified that the susceptibility $\left. \frac{\partial^2 \mathcal{E}}{\partial \Lambda^2} \right|_{s=constant}$ is positive — Λ is the scale introduced by the relevant operator \mathcal{O}_r , see [17] for more details.

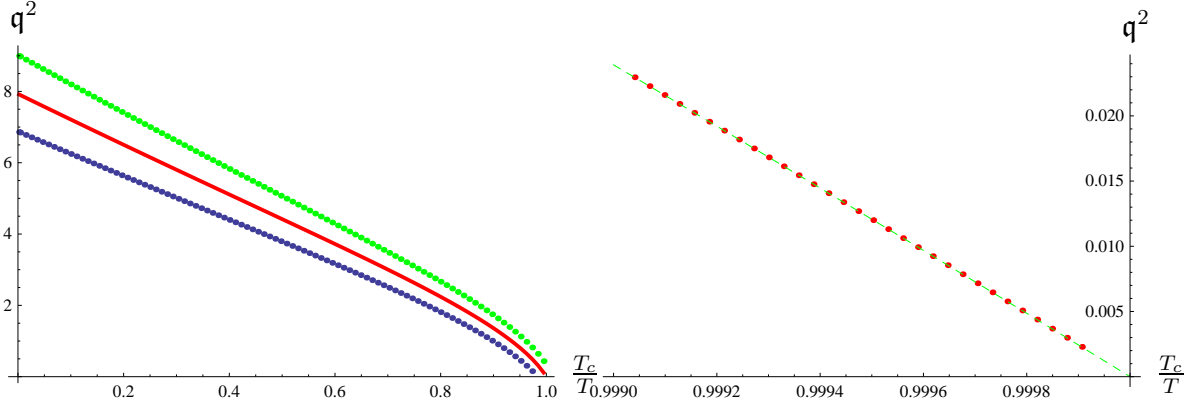


Figure 3: (Colour online) Dispersion relation of quasinormal modes in the symmetry broken (classically unstable) phase of the exotic black branes. The red line (left plot) represents the modes at the threshold of GL instability: $(\mathfrak{w}, \mathfrak{q}) = (0, \mathfrak{q}_c)$. The green/blue dots (left plot) are the stable/unstable quasinormal modes with $\mathfrak{w} = -0.1i$ and $\mathfrak{w} = 0.1i$ correspondingly. The right plot presents the modes at the threshold of instability in the vicinity of the critical point. The dashed green line (right plot) is the best quadratic in $(1 - \frac{T_c}{T})$ fit to data.

The dispersion relation for the sound mode takes the form

$$\mathfrak{w} = \pm c_s \mathfrak{q} - \frac{i}{4} \left(1 + \frac{\zeta}{\eta} \right) \mathfrak{q}^2 + \mathcal{O}(\mathfrak{q}^3), \quad (2.6)$$

implying that the sound modes are stable as well.

Since the free energy of the symmetry broken phase is bigger, this phase is not thermodynamically preferable and one expects a classical instability driving the system to a true (symmetric) ground state. Such instability is expected to be of GL type, see (1.1), and thus is not expected to show up in the hydrodynamic limit. In the next section we explicitly identify the Gregory-Laflamme instability, and thus prove that CSC is false.

3 GL tachyons of exotic black branes

Physical excitations in a gauge theory plasma are dual to quasinormal modes of a corresponding black brane solution [23]. For a translationally invariant horizon, a wave-function of a generic background field fluctuation $\delta\Phi$ takes form

$$\delta\Phi = \mathcal{F}(r) e^{-i\omega t + i\vec{k} \cdot \vec{x}}, \quad (3.1)$$

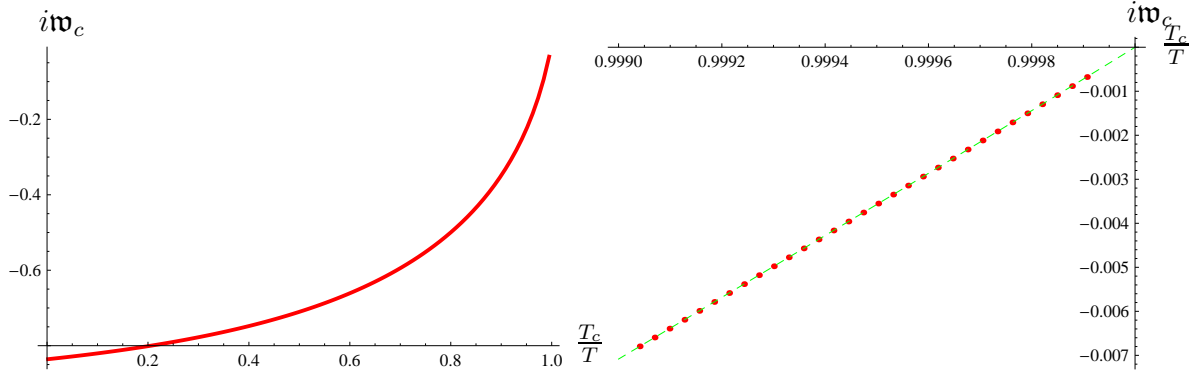


Figure 4: (Colour online) Frequency of the homogeneous and isotropic unstable quasinormal modes $(\mathfrak{w}, \mathfrak{q}) = (\mathfrak{w}_c, 0)$ in the symmetry broken (classically unstable) phase of the exotic black branes. The dashed green line (right plot) is the best quadratic in $(1 - \frac{T_c}{T})$ fit to data in the vicinity of the critical point.

where \mathcal{F} is a radial wave function. Fluctuations of different fields will typically mix with each other. It is convenient to decompose the fluctuations in irreducible representations with respect to rotations about the \vec{k} -axis. In the case of the exotic black branes, the GL instabilities are expected to arise as fluctuations in the symmetry-breaking (scalar) condensate. Thus, they appear in the *scalar* quasinormal mode channel. It is exactly the same set of fluctuations which in the hydrodynamic limit ($\mathfrak{w} \rightarrow 0$, $\mathfrak{q} \rightarrow 0$ and $\frac{\mathfrak{w}}{\mathfrak{q}} \rightarrow \text{constant}$) describe the propagation of the sound waves in plasma. The technique for computing the scalar channel quasinormal modes in non-conformal holographic models was developed in [24]. It is straightforward to generalize the method of [24] to the computation of the scalar channel quasinormal modes of the gauge theory plasma dual to the holographic RG flow [17]. The details of the latter analysis will appear elsewhere [25] and here we report only the results, focusing on the GL mode⁶.

The red solid line (left plot) in Figure 3 presents the dispersion relation for the quasinormal modes in the symmetry broken (classically unstable) phase at the threshold of instability:

$$(\mathfrak{w}, \mathfrak{q}) \Big|_{\text{threshold}} = (0, \mathfrak{q}_c), \quad (3.2)$$

as a function of the reduced temperature $\frac{T_c}{T}$. The modes with $\mathfrak{q}^2 > \mathfrak{q}_c^2$ attenuate, *i.e.*, they have $\text{Im}(\mathfrak{w}) < 0$, while those with $\mathfrak{q}^2 < \mathfrak{q}_c^2$ represent a genuine GL instability —

⁶Sound waves were discussed in [18] and reviewed in the previous section.

they have $\text{Im}(\mathfrak{w}) > 0$. The green dots on the left plot have $\mathfrak{w} = -0.1i$ and the blue dots have $\mathfrak{w} = 0.1i$. The right plot on Figure 3 represents the dispersion relation of the quasinormal modes in the symmetry broken phase at the threshold of instability in the vicinity of the critical point. The green dashed line is the best quadratic in $t \equiv (1 - \frac{T_c}{T})$ fit to the data:

$$\mathfrak{q}_c^2 \Big|_{fit} = 3.(4) \times 10^{-5} + 24.(3) t - 369.(1) t^2 + \mathcal{O}(t^3). \quad (3.3)$$

The data suggests that

$$\mathfrak{q}_c^2 \propto (T - T_c), \quad (3.4)$$

in the vicinity of the critical point.

The left plot in Figure 4 represents the unstable homogeneous and isotropic quasinormal modes, *i.e.*, GL instabilities with $\mathfrak{q} = 0$. The right plot shows the frequency dependence of these modes in the vicinity of the critical point. The green dashed line is the best quadratic in t fit to the data:

$$i\mathfrak{w}_c \Big|_{fit} = -9.(8) \times 10^{-6} - 7.(2) t + 102.(9) t^2 + \mathcal{O}(t^3). \quad (3.5)$$

The data suggests that

$$\mathfrak{w}_c \propto i(T - T_c), \quad (3.6)$$

in the vicinity of the critical point.

Notice that since $\mathfrak{w}_c \propto i\mathfrak{q}_c^2$ in the vicinity of the critical point, it is natural to identify the dynamical critical exponent of the model with $z = 2$. The symmetries of our model identify it as 'model A' according to classification of Hohenberg and Halperin (HH) [26], assuming a natural extension of the classification to classically unstable phases. Mean-field models (with vanishing anomalous static critical exponent) in the HH universality class A are predicted to have the dynamical critical exponent $z_{model-A} = 2$, as the one we obtained⁷.

In this section we explicitly identified the Gregory-Laflamme instabilities in the symmetry broken phase of the exotic black branes. Since this phase is thermodynamically stable, our model presents a counter-example to the correlated stability conjecture.

⁷The second-order phase transition in exotic black branes is of the mean-field type [25].

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